The second term in the brackets of (13) is positive definite for $e < 1$ and has a zero limiting value as $e \to 0$ In a similar way, the first-order solution of φ is found as $\varphi_1 = \bar{\varphi}_1(\Theta + \varphi_0)$, where

$$\bar{\varphi}_1 = -\left\{\left(1 + \frac{2\mu I_y'}{\bar{\delta}}\right)\frac{1}{(1+e)} + \frac{1}{4}\frac{e^2}{(1+e)}\frac{I_y'\mu}{\bar{\delta}} + \frac{1}{e}(1+e)^2(1-e)^3 \times (I_{y_1}'G_1 + I_{y_2}'G_2)\frac{\mu}{\bar{\delta}}\left[(1-e^2)^{-3/2} - \frac{1}{e^2}\left(\frac{2-e^2}{(1-e^2)^{1/2}} - 2\right)\right]\right\} \quad (14)$$

By substituting $A = A_0 + \epsilon A_1(\Theta)$ and $\varphi = \varphi_0 + \epsilon\varphi_1(\Theta)$ into (10) and changing the variables back to r, the perturbed orbit resulting from the steady-state forced librational motion is obtained Note that the initial phase angle φ_0 can be taken to be equal to zero by transformation of the coordinates O-XYZ

Orbital decal rate can be found from the expression of the semimajor axis $r_0(\Theta)$ of the orbital ellipse, which is one-half the sum of the two apsidal distances, corresponding to $\cos(1 + \epsilon\bar{\varphi}_1)\Theta = \pm 1$, i e , $r_0(\Theta) = r_0 + \Delta r_0(\Theta)$, where

$$\Delta r_0(\Theta) = -\frac{4c(P_1 - P_2)^2}{r_0\Omega m\Delta}e^2\left[(1-e^2)^{-3/2} + \frac{1}{e^2}\left(\frac{2-e^2}{(1-e^2)^{1/2}} - 2\right)\right] \quad (15)$$

Orbital phase shift is simply

$$\epsilon\varphi_1 = -\left\{\frac{(\bar{\delta}/\mu + 2I_y')}{r_0^2(1-e^2)^2} + \frac{e^2}{4(1-e^2)^2}\frac{I_y'}{r_0^2} + \frac{1}{r_0^2}\left(I_{y_1}'\frac{G_1}{e} + I_{y_2}'\frac{G_2}{e}\right)(1-e^2) \times \left[(1-e^2)^{-3/2} - \frac{1}{e^2}\left(\frac{2-e^2}{(1-e^2)^{1/2}} - 2\right)\right]\right\}(\Theta) \quad (16)$$

Numerical Example

To illustrate numerically the magnitude of the rates of orbital decay and phase shift, let us take a two-body satellite with the following parameters, which has been demonstrated in Refs 4 and 7 to be feasible in gravitational orientation: $m = 10$ slugs, $I_{x1} = I_{y1} = 3300$, $I_{z1} = 10$, $I_{x2} = 450$, $I_{y2} = 1000$, $I_{z2} = 1450$ slug-ft², coefficient of viscous friction $c = 4270\Omega$ ft-lb-sec, spring constant $k = 4300\Omega^2$ ft-lb/rad Hence, $I_y' = 430$ ft², $\bar{\delta}/\mu \approx 775$ ft², $P_1 = 2$, $P_2 = -4$, $\bar{c}_1 = 1\ 28$, $\bar{c}_2 = 4\ 27$, $k_1 = 1\ 29$, $\bar{k}_2 = 4\ 3$, $\Delta = 32\ 5$, $G_1 = -2\ 82e$, $G_2 = -4\ 7e$ Then, for 1000-mile alt or $r_0 = 2\ 62 \times 10^7$ ft ($\Omega = 0\ 89 \times 10^{-3}$ rad/sec) and $e = 0\ 1$ (note: from the work of Refs 4 and 7, e should be kept smaller than 0 15 for the librational motion to be valid), it is computed from (15) that the rate of orbital decay is $\Delta r_0/\Delta t = 0\ 02$ ft/yr and from (16) that the rate of apsidal advance is $\Delta\varphi/\Delta t = 0\ 65 \times 10^{-6}$ deg/yr

The orbital decay rate can be estimated in another way from the formula $\Delta r_0 \approx (m\mu/2)(\Delta E/E^2) = (2/gm)(r_0/R_0)^2\Delta E$, where $E = -\mu m/2r_0$ is the total orbital energy, and

$$\Delta E \approx \int_0^{2\pi/\Omega} c(\dot{\theta}_1 - \dot{\theta}_2)^2 dt = \pi c\Omega[(F_1 - F_2)^2 + (G_1 - G_2)^2]$$

is the energy dissipation per cycle through the steady-state forced librational motion From this, it is calculated that $\Delta r_0 \approx 0\ 024$ ft/yr, which checks fairly well with the result given in (15) The same type of approximation applied to a magnetically oriented satellite should produce results of comparable accuracy

From the preceding numerical results it is concluded that these long-term effects are negligibly small for a gravitationally oriented satellite It may be of interest, however, to attempt an application of the foregoing formulas to astronomical problems

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Energy Solution for Simply Supported Oval Shells

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Introduction

A RECENT study of clamped, short, oval shells under hydrostatic load[1] presented an energy solution of the shell equations The present note shows the results of applying this solution to simply supported, short, oval shells under constant lateral load The shells considered have major-to-minor axis ratios b/a in the range 1 00 (circular) $\leq b/a \leq$ 2 06 For values of $b/a >$ 2 06, the assumed shell cross section [see Eq (1)] does not remain convex at all points

The results are compared to a double Fourier series solution, which is presented in Refs 2 and 3 for values of b/a of 1 10, 1 51, and 2 06 and which is herein considered to be exact For other values of b/a an equivalent circular shell solution, which in Refs 2 and 3 is shown to be in good agreement with the exact solution, has been used as a basis for comparison This approximate solution is obtained by using the value of the local radius of curvature of the oval shell for the radius in the well-known axisymmetric solution of the circular, cylindrical shell equations

The energy solution used can be applied to a shell having arbitrary boundary conditions (e g , a shell with elastic ring supports), whereas the double Fourier series or equivalent circular shell solutions are not readily applicable to such a shell

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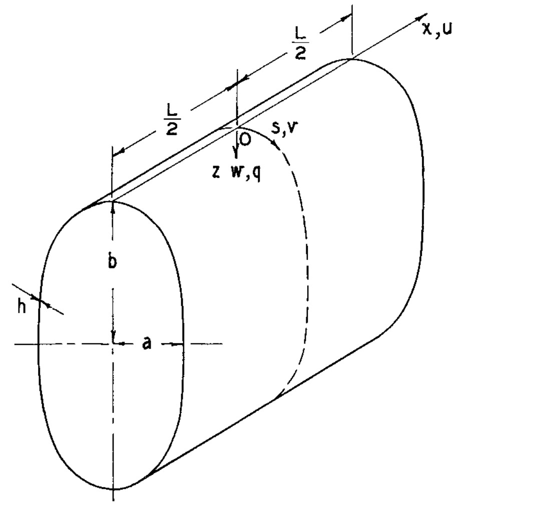

Fig 1 Sign convention for coordinates and displacements

Assumptions and Method of Solution

As in Refs 1–4, the local curvature $1/r$ of the median line of the oval shell cross section (see Fig 1) is assumed to be given by

$$(1/r) = (1/r_0)[1 + \xi \cos(4\pi s/L_0)] \quad (1)$$

where L_0 is the oval perimeter, $r_0 = L_0/2\pi$, and ξ is a parameter that fixes b/a The variation of b/a with ξ is discussed in Refs 2 and 3 The complete shell geometry is fixed by additionally specifying the axial length L and wall thickness h Any point of the shell is located by specifying its axial, circumferential, and radial coordinates, x, s, and z, respectively The cross section characterized by Eq (1) is symmetric with respect to both the major and minor axes of the oval Therefore, under the action of a constant lateral load q_0, the axial, circumferential, and radial displacements, u, v, and w, respectively, are such that the deformed oval is similarly symmetric

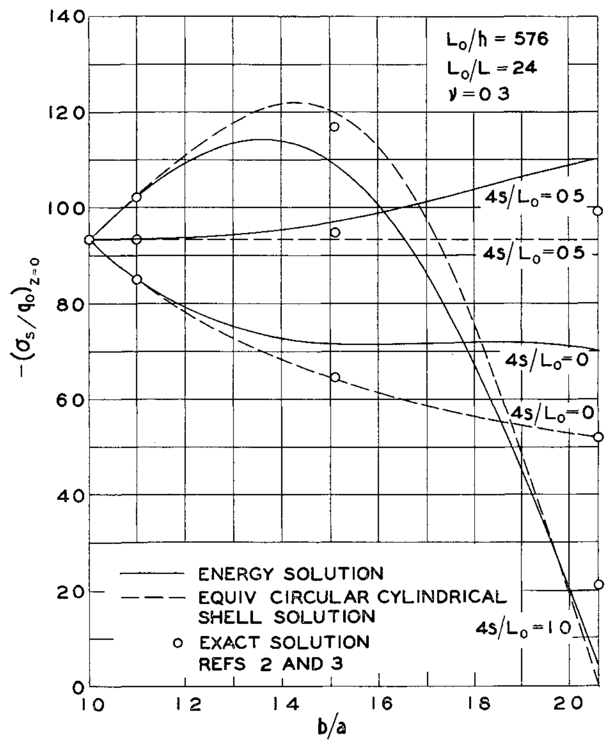

Fig 2 Circumferential membrane stress vs b/a for values of $4s/L_0$ at $2x/L = 0$

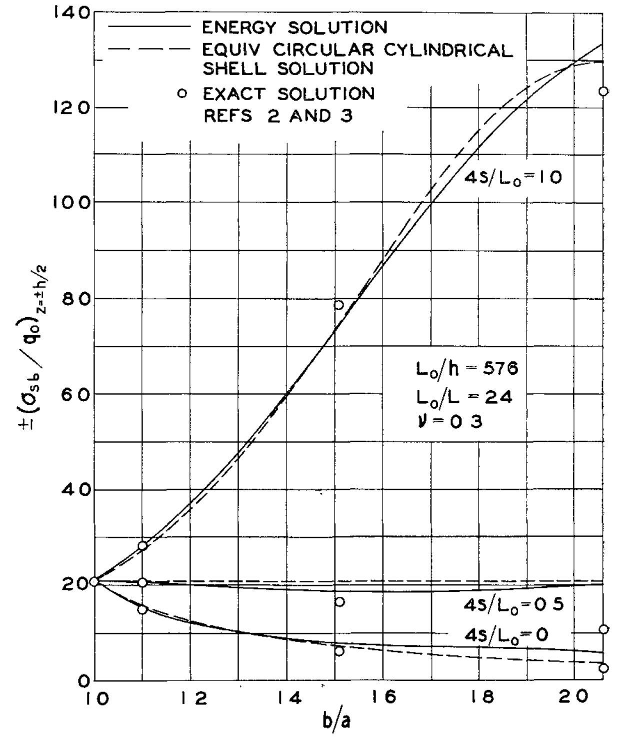

Fig 3 Circumferential bending stress vs b/a for values of $4s/L_0$ at $2x/L = 0$

In order to apply the energy solutions developed in Ref 1, u, v, and w are expressed as a truncated Fourier series in s Only the fundamental, first, and second harmonics are retained in these series, which have the period $I_0/2$ in s and reflect the symmetry of the deformed oval The eight coefficients of the series (three each for u and w, and two for v) are assumed to be arbitrary functions of x A set of ordinary differential equations as well as the associated natural boundary conditions for the determination of the unknown displacement coefficients are obtained by applying the principle of the minimum of the total potential The details for solving these equations for a simply supported oval shell are given in Ref 4 The strain-displacement relations used in the energy theorem correspond in accuracy to those of

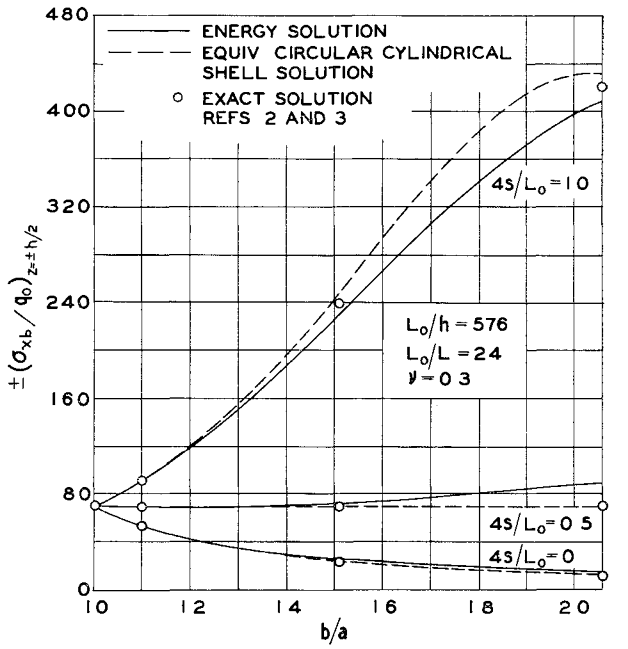

Fig 4 Axial bending stress vs b/a for values of $4s/L_0$ at $2x/L = 0$

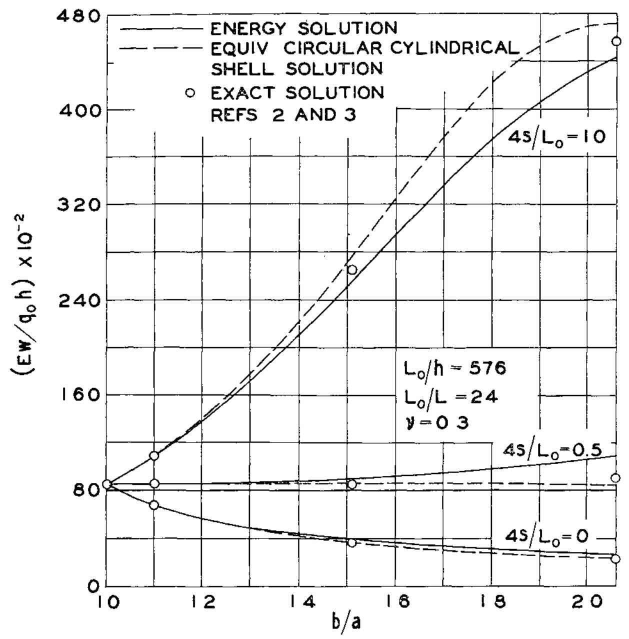

Fig 5 Radial displacements vs b/a for values of $4s/L_0$ at $2x/L = 0$

Donnell for circular, cylindrical shells if nonlinear (buckling) terms are omitted The stresses are related to the strains by Hooke's law for plane stress

Discussion

Numerical results have been obtained for oval shells in which $L_0/h = 576$ and $L_0/L = 24$ Poisson's ratio ν was taken as 0 3; Young's modulus E is arbitrary Figures 2–5 show the variation with b/a of the nondimensional membrane stress $(\sigma /q_0)_{z=0}$, circumferential bending stress $\pm(\sigma_b/q_0)_{=\pm h/2}$, axial bending stresses $\pm(\sigma_{xb}/q_0)_{z=\pm h/2}$, and radial displacement (Ew/q_0h) Each of these quantities is shown at mid-bay $(2x/L = 0)$ for the three generators $(4s/L_0) = 0$ (major axis), 0 5 (at which $r = r_0$), and 1 0 (minor axis)

For $b/a = 1$ 10, 1 51, and 2 06, respectively, the maximum circumferential stress, a compressive stress given by $[\sigma_s/q_0]_{max} = [(\sigma /q_0)_{z=0} + (\sigma_b/q_0)_{=-(h/2)}]_{max}$, varies at most by $\frac{1}{10}$%, 6%, and 11% from the exact solution For the same values of b/a the maximum values of the axial bending stress are in error by $\frac{1}{20}$%, 4%, and 7% Similarly, the maximum radial displacements differ by $\frac{1}{10}$%, 3%, and 3% for the exact solutions These results, together with those of Ref 1, suggest that the energy solution used can be applied with some degree of confidence to boundary conditions other than those considered herein and in Ref 1

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Range and Angle Prediction Tracking of Objects with Definable Trajectories

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Introduction

THE usual prediction-tracking equations employed in most radar systems are a form of a Taylor's series in the time variable This type of prediction-tracking equation is written as

$$X_i = X_{i0} + \dot{X}_{i0}(t - t_0) + \ddot{X}_{i0}(t - t_0)^2/2 +$$

where X_i is a position coordinate of interest at time t predicted from the position, velocity, and acceleration data obtained at time t_0 Since the third time derivative of the position coordinate (the jerk function) is difficult to evaluate from radar data, this type of prediction equation is usually truncated after the quadratic term As a result of truncation errors, the Taylor's series form of prediction is limited to a time interval of the order of a second or less In addition to the truncation errors of the prediction equation, the method of prediction by storing ground coordinates (Cartesian coordinates with the origin at the radar site) involves coordinate transformation errors In the case of the radar range variable, the coordinate transformation errors are of paramount significance The radar range is the most accurate data obtainable from a short-pulsed radar signal Relative to the radar range resolution, the angular resolution of the target's data is extremely crude Thus, by performing a coordinate transformation from the radar range-angle coordinates to the ground coordinates, the accuracy of the radar range is diluted by the errors in the angular data

As a consequence of the propagation of errors through coordinate transformation, the natural solution is to attempt to store data and predict the target's behavior in the radar range-angle coordinate system Since the Taylor's series time predictor is inadequate for the forementioned reasons, a new approach was investigated for a completely general range-angle predictor The results of the analysis are applicable to ballistic missile trajectories, satellite trajectories, nonmaneuvering aircrafts, and any trajectory that lies in a given plane and whose motion in that plane is predictable or obeys a set of physical laws

The radar range tracking equation is an exact closed-form solution for the radar range in terms of the target parameters and is independent of the radar angle requirements for range-only tracking Angle tracking equations are also given in terms of the radar range and the target parameters

Assumptions

In the derivation of the range prediction-tracking equation, the analysis is based upon a spherical earth geometry The approximation is valid within the specified radar coverage volume since, for any given radar site, the oblateness effects can be taken into account by matching a spherical earth model to the known direction and magnitude of the radar site gravity vector Furthermore, the analysis is based upon a radar tracking station located on the surface of a nonrotating earth For practical applications of a radar site on a rotating earth at any given latitude, equations are easily generated for the rotating earth corrections

No assumptions are made with respect to the method in which the trajectory is defined, with the exception that it is

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